**Electric Dielectrics**

So I guess we’re doing the electric susceptibility stuff. We’ll presume we’re in a time-independent situation, and so everything has come to equilibrium. Or possibly we could allow time-dependence, as long as it is much slower than the thermal relaxation time. In this case, time would just be a spectator variable. The formalism below is found in the EM folder too.

**Electric Susceptibility**

So the conceit is that we introduce a free charge/current density, which gives rise to free fields, and ultimately induces a responsive charge/current density. I’ll redo the analysis we did in EM file for metals, because of the slightly different pre-factors we’re using, mainly due to fact we’re in fake ‘Gaussian’ units now [basically can go from SI to fake Gaussian by ε0 → 1/4π, μ0 → 4π]. I’ll also allow time-dependent fields/susceptibilities, so we don’t have to redo the whole thing in the non-equilibrium file. Referring to the Units file, if we were to take the electric susceptibility equation:



and convert to fake Gaussian units, we’d get:



But instead, we’re going to write:



So this χirr is smaller than the χirr we defined in the EM folder by a factor of 4π, and the ‘advantage’ of doing so is it allows us to put things more easily in terms of the Coulomb potential’s Fourier transform. Anyway, this is equivalent to, taking the spatial/temporal Fourier transform of both sides:



Now how do all of these functions relate? We can use Maxwell’s equations, like was done before. We’ll use:



Taking the Fourier transform of both sides,



Plugging in our induced ρ,



And now we can define the proportionality between ρf and φ as:



where υ(q) = 4π/q2. This is called υ(q) because the Coulomb potential has the same Fourier transform, in Gaussian units, sans the factors of e. Must be careful not to confuse this υ(q) with the actual potential V(q) that *includes* the e’s. Now let’s relate the induced charges to the free potential. We have by definition:



and for the other, we have:



Now we’ll use this to define χ(q):



There are lots of other ways we can write χ(q)…



For reference down below, we’ll observe that the last equality implies ε = 1/(1+υχ). Finally, observe we can relate ρind to ρf via:



and we can put the total field in terms of the free field,



So,



**χ(q) in terms of GF’s**

There is another interesting way to do this, presented by Mahan. And in the process we’ll get an expression for χ(q) in terms of GF. Consider that we place two external charges Z1e and Z2e in an otherwise homogeneous electron gas. Then the unscreened potential energy between them would be:



The fields from the two charges will induce a charge density which will screen the external fields. The new potential energy we can write as:



[I’m not sure this follows exactly, but…] We can calculate the screened potential energy in the following way. We consider the Hamiltonian of our system.



H012 is the Hamiltonian for the homogeneous electron gas – including kinetic energy, potential energy, etc. ρ(q) is the density operator for the electron gas. υ(q) = 4π/q2, the Fourier transform of the electrostatic potential energy, sans the e2. The ground state energy can be calculated and will include terms proportional to Z1Z2. We will collect these terms and combine them with Vext. The net result we’ll set equal to:



and in this way extract the dielectric function. The ground state energy is calculated from the thermodynamic potential, which is found via the cluster expansion (see Stat Mech/Quantum Perturbation Theory file).



and, going back to position space, our perturbation is:



We only need to carry out the expansion to 2nd order since only up to this point do we get cross terms proportional to Z1Z2. The result is:



where we define/recognize (n being number particle density),



[in this scenario n = δn since n0 = 0?] We only need the cross terms to get ε, so just keeping these, eventually we should get:



Now can translate this in terms of FT,



And so, the ground state energy part that contains the Z1Z2 factors is (remember U2 enters into the energy with a -1/β factor):



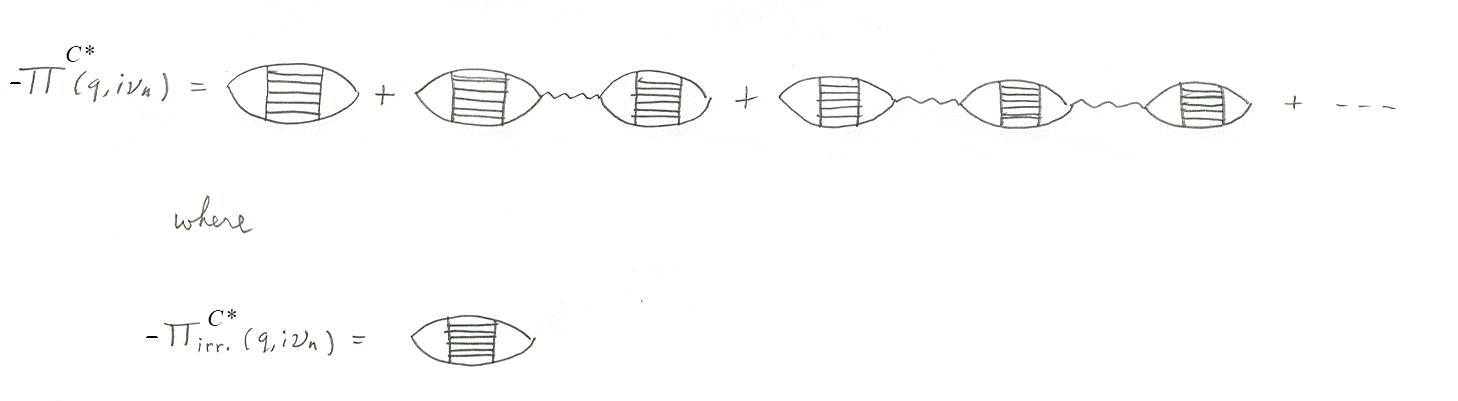
So we have,



We’ll see this generalizes quite well to non-zero frequences, in the non-equilibrium file. This allows us to identify



(see highlighted formulas in the first section). We can go back to the collective excitations file and recall that we can write ΠC\*(q) in terms of its self energy expansion,



which amounts to:



Filling this in, we have:



So,



which allows us to identify



Again, see highlighted formulas in the first section.